LINEAR INEQUALITIES

Basics of Inequalities

- $a > b \Rightarrow a + c > b + c$

- $a > b \Rightarrow \begin{cases} ac > bc, \text{if } c > 0 \\ bc > ac, \text{if } c < 0 \end{cases}$

  means, $\mathbf{a > b \Rightarrow -a < -b}$

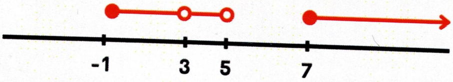
- $ab > 0$, if a, b have same signs
  $ab < 0$, if a, b have opp. signs

- $a > b \Rightarrow \begin{cases} \dfrac{1}{a} < \dfrac{1}{b}, \text{if } ab > 0 \\ \dfrac{1}{a} > \dfrac{1}{b}, \text{if } ab < 0 \end{cases}$

Square and square roots of inequalities are not done

Example of Representation on lines

$x \in [-1,3) \cup (3,5) \cup [7,\infty)$



● Includes   ○ Doesn't Include

WAVY CURVE METHOD

When powers of factors are not included

Step 1	Put all factors equal to zero & mark corresponding values on real line
Step 2	Put +ve on extreme right & alternate signs towards left

$$\frac{(x-1)(x-3)}{(x-4)} \geq 0$$

Put factors = 0, we get,
x = 1 or x = 3 or x ≠4

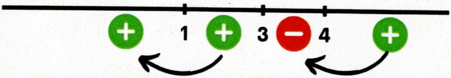
We need greater or equal to 0. So, **x ϵ [1,3]∪(4,∞)**

When powers of factors are included

- For odd powers, change sign
- For even Powers, Continue same sign

$$\frac{(x-1)^{34}(x-3)^{15}}{(x-4)^{27}} \geq 0$$

Put factors = 0, we get,
x = 1 or x = 3 or x ≠4



Sign **Remains same** as (x-1) has an even power

Sign **changes** as (x-4) has an odd power

So, **x ϵ (-∞,3]∪(4,∞)**

Modulus Related Equations

- $\sqrt{x^2} = |x| = \pm x$
- $|x| = a$ then $x = \pm a$
- $|x| \leq a$ then $x \epsilon [-a, a]$
- $|x| \geq a$ then $x \epsilon (-\infty, a] \cup [a, \infty]$
- $a < |x| < b$ then $x \epsilon (-b, -a) \cup (a, b)$
- $a < |x - c| < b$
  then $x \epsilon (-b + c, -a + c) \cup (a + c, b + c)$

Logarithmic Inequality

For $\log_a b$, then following should be remembered

Condition	Logarithm Inequality	Exponential form
a > 1	$\log_a x_1 > \log_a x_2$ $\Rightarrow x_1 > x_2$	$\log_a b \geq c$ $\Rightarrow b \geq a^c$
0 < a < 1	$\log_a x_1 > \log_a x_2$ $\Rightarrow x_1 < x_2$	$\log_a b \geq c$ $\Rightarrow b \leq a^c$

Exponential Inequality

Condition	Exponential form
a > 1	$a^{x_2} > a^{x_1} \quad \Rightarrow x_1 > x_2$
0 < a < 1	$a^{x_2} > a^{x_1} \quad \Rightarrow x_1 < x_2$